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Validation of the Inverse Method of Acoustic Material Characterization

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ABSTRACT

There are many software tools in use today that are implementing the Biot, or complementary, method for the evaluation of foam and fiber materials. The justification of this process is to understand which mechanisms of the noise control material are contributing to the noise reduction and to optimize the material based on its acoustic properties. The disadvantage of this method is that it is quite complex and time consuming to test a material in order to extract the underlying properties that govern the acoustic performance. An alternative inverse method for material characterization based on simple impedance tube measurements has been developed lately. This paper recalls the physics and mathematics behind the method and concentrates on its validation.

INTRODUCTION

Open-cell acoustical materials such as open foam, glass fibers, and felts (see Figure 1) are commonly used for their sound absorbing properties. When excited by acoustic waves these materials can be approximated to behave as acoustically rigid over a wide range of frequencies. In this case the acoustic wave, propagating in the air and saturating the porous network, is mainly attenuated through viscous and thermal losses. Scientists have shown that these losses are mostly related to five geometrical properties of the porous medium; flow resistivity (σ), porosity (ϕ), tortuosity (α_∞), viscous (Λ) and thermal (Λ') characteristic lengths [1]. While the first two properties can be measured without great difficulties, the remaining are substantially more difficult to measure. Yet the latter also represent a significant factor in the performance of acoustical materials.

To circumvent these limitations and to minimize the requirement for characterization tests, an alternative inverse method has been developed [2]. This inverse method is based on standard measurements using an impedance tube [ASTM E 1050]. This paper recalls the physics and mathematics behind the method, and concentrates on its validation.

Figure 1 : Examples of acoustical materials

THEORY

Given a set of acoustical observed data (Φ_i), we want to condense and summarize the data by fitting them to a model, which depends on adjustable parameters, namely porosity (ϕ), tortuosity (α_∞), resistivity (σ), viscous characteristic length (Λ), and thermal characteristic length (Λ'). To do so, a merit function is designed to measure the agreement between the observations and the model predictions ($\Phi(\omega_i;\mathbf{a})$), with a particular choice of a parametric vector $\mathbf{a}$. In the developed method, the parametric vector $\mathbf{a}$ for a five-parameter identification is $\mathbf{a}=\{\phi,\sigma,\alpha_\infty,\Lambda,\Lambda'\}$ and for a three-parameter identification $\mathbf{a}=\{\alpha_\infty,\Lambda,\Lambda'\}$. The parameters of the model are then adjusted to reach a minimum in the merit function, yielding best-fit parameters. The adjustment process is thus a problem of minimization of the merit function in a five or three dimensions space.

ACOUSTIC MODEL

The model in question is nonlinear and is based on an observable acoustic parameter, which is the sound absorption coefficient of a material backed by a rigid wall given by:

$$\Phi_i = \Phi(\omega_i;\mathbf{a}) \qquad (1)$$

where Φ is the measured value at the ith angular frequency ω_i, $\Phi(\omega_i, \mathbf{a})$ is the corresponding prediction for the set **a**. Following the porous material model [2], the mathematical expression for the sound absorption coefficient is given by:

$$\Phi(\omega_i;\mathbf{a}) = 1 - \left| \frac{Z(\omega_i;\mathbf{a}) - 1}{Z(\omega_i;\mathbf{a}) + 1} \right|^2 \tag{2}$$

where the surface impedance of a porous layer of thickness *h*, backed by a rigid wall, is

$$Z(\omega_i z;\mathbf{a}) = j\frac{1}{\phi Z_0}\sqrt{\rho_e(\omega) K_e(\omega)} \cot\left(\omega h \sqrt{\frac{\rho_e(\omega)}{K_e(\omega)}}\right) \tag{3}$$

where Z_0 is the characteristic impedance of air. ρ_e and K_e are the dynamic density and dynamic bulk modulus of the air in the pores, respectively. They are given by:

$$\begin{aligned} \rho_e(\omega) &= \left(\frac{1}{\rho_0 \alpha(\omega)} + \mathrm{B} \cdot \frac{(\phi/\alpha(\omega) - 1)^2}{\phi\rho_1 + \phi^2 \rho_0 (1 - 1/\alpha(\omega))} \right)^{-1} \\ K_e(\omega) &= \frac{\gamma P_0}{\gamma - (\gamma - 1)\left(1 - j\frac{H'}{2\omega}\sqrt{1 + j\frac{\omega}{H'}}\right)^{-1}} \end{aligned} \tag{4}$$

with

$$\alpha(\omega) = \alpha_\infty \left(1 - j\frac{\phi\sigma}{\omega\rho_o\alpha_\infty}\sqrt{1 + j\frac{\omega}{H}}\right),$$

$$H = \frac{\sigma^2 \Lambda^2 \phi^2}{4\alpha_\infty^2 \eta \rho_0} \quad \text{and} \quad H' = \frac{16\eta}{\mathrm{Pr}^2 \Lambda'^2 \rho_0}$$

where ρ_0 is the air density, η the dynamic viscosity of air, ρ_0 the bulk density of the frame, γ the specific heat ratio, Pr the Prandtl number, and P_0 the barometric pressure. Coefficient B appearing in the dynamic density is equal to 0 if the frame of the material is "rigid", or equal to 1 if the frame is "limp".

For most sound absorbing materials, the following bounds apply on the adjustable parameters:

$$\begin{aligned} & 1 \le \alpha_\infty \le 4 \\ & \tfrac{1}{3.3}\left(\tfrac{8\alpha_\infty \eta}{\sigma\phi}\right)^{1/2} \le (\Lambda, \Lambda') \le \tfrac{1}{0.3}\left(\tfrac{8\alpha_\infty \eta}{\sigma\phi}\right)^{1/2} \\ & \Lambda \le \Lambda' \\ & \sigma > 0 \\ & 0 < \phi < 1 \end{aligned} \tag{5}$$

The dynamic density and bulk modulus in eqn (4) are complex functions, which heavily depend on the angular frequency ω. They also take into account the inertial interaction between the fluid and solid phases of the porous material, as well as the viscous and thermal losses.

MERIT FUNCTION

As mentioned previously, the basic approach of the inverse characterization is to identify the parametric vector **a** that will minimize a merit function built from observations and predictions. Here the merit function that is considered is the chi-square statistic χ^2, which arises in a slightly different context from its general definition. It is given by:

$$\chi^2(\mathbf{a}) = \sum_{i=1}^{N} \left[\left(\Phi_i - \Phi(\omega_i;\mathbf{a})\right)_{h_1}^2 + \left(\Phi_i - \Phi(\omega_i;\mathbf{a})\right)_{h_2}^2 \right], \tag{6}$$

where Φ_i is the acoustical indicator measured at ω_i. The first term in eqn (6) is associated to a specimen of thickness h_1, while the second term is associated to a specimen of thickness h_2.

To find the best-fit parametric vector **a,** one needs to find the global minimum of the merit function in eqn (6). This is accomplished with a minimization algorithm, which has not been derived in this paper. The set-up of this algorithm is beyond the scope of this paper and the reader should refer to [2].

RESULTS

RIGID FOAM MATERIAL

Using the inverse method, a rigid metal foam is first characterized; B=0 in eqn (4). Two 9.56-mm thick specimens are used (see Figure 2). The absorption of a single specimen and a stack of two specimens are measured using a 29-mm impedance tube between 300 and 6000 Hz. Applying the inverse procedure, the unknown parametric vector **a** is computed to best-fit the two experimental sound absorption curves. The results of the minimization are shown in Figure 3.

Now, to ensure that the parametric vector is not only a mathematical optimum, its 5 coefficients are compared to classical direct characterization methods [3-6]. The comparison is given in Table I. One can note that the parameters are in good correlation to the direct measurements.

LIMP FIBER MATERIAL

The second material to characterize is a limp fiber material; B=1 in eqn (4). The configuration shown in Figure 2 is used once again; however this time the thicknesses are 27 and 54 mm. The porosity and resistivity of the material measured using the direct methods [3-5] are 0.89 and 42 500 Ns/m^4, respectively.

For the inverse method, the three-parameters and the five-parameters identifications are compared. The three-parameters identification uses the directly measured porosity and resistivity. For both identifications, the minimization of eqn (6) is performed using the 500-4000 Hz frequency range. The solutions of the minimization are given in Table II. It is noted that both identifications yield similar properties and compare well with the direct results.

Table I – Comparison between direct and inverse measurements of the rigid metal foam properties.

	Direct	Inverse (5-p)
Porosity	0.89 ± 0.02	0.885
Resistivity (Ns/m⁴)	43 456 ± 1 257	41 250
Tortuosity	1.13 ± 0.04	1.14
Viscous length (µm)	17 ± 2	17
Thermal length (µm)	131 ± 8	137

Table II – Comparison between direct and inverse measurements of the limp fiber material properties.

	Direct	Inverse (3-p)	Inverse (5-p)
Porosity	0.89 ± 0.01		0.89
Resistivity (Ns/m⁴)	42 500 ± 320		42 035
Tortuosity	1.01 ± 0.03	1.00	1.00
Viscous length (µm)	51 ± 15	46	33
Thermal length (µm)	88 ± 17	74	87

Figure 2 – (a) Two metal foam specimens. (b) A stack of the two specimens.

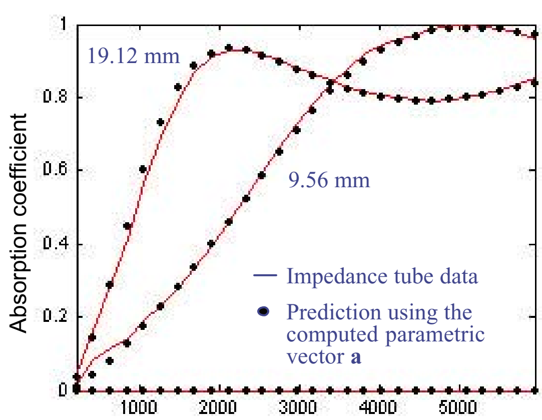

Figure 3 – Sound absorption coefficient. Measurements versus predictions. The predictions are obtained with eqn (2) and the optimal parametric vector **a**.

DISCUSSION AND CONCLUSION

This paper has shown that an inverse acoustical identification method, based on standardized impedance tube measurements, is a promising alternative to direct measurements of the acoustical properties of foam and fiber materials. It was shown that the method gives good estimates of the intrinsic properties of the tested materials and it has been found that the values, obtained by the inverse method, can be more accurate than the direct measured results.

However, care is required with the inverse method in choosing the appropriate frequency range used for inversion, and to minimize errors on the directly measured parameters (thickness, density, resistivity, porosity). As a first example, if a reduced frequency range (300-1600 Hz) is used with the three-parameters identification of the limp fiber material, the set of parameters found is $\{\alpha_\infty, \Lambda, \Lambda'\} = \{1.1, 26\mu m, 124\mu m\}$. As a second example, assume a porosity 5 % greater than the actual one for the limp fiber material (0.93 instead of 0.89). This time, the three-parameters identification yields the set of parameters $\{\alpha_\infty, \Lambda, \Lambda'\} = \{1, 37\mu m, 37\mu m\}$. If the error is on the thickness (30 instead of 27 mm), the set of properties found is $\{\alpha_\infty, \Lambda, \Lambda'\} = \{1, 53\mu m, 345\mu m\}$. In all these examples, the errors on the found parameters are large compared to the direct values.

Further investigations are necessary to minimize and quantify the errors of the inverse method on different types of sound absorbing materials. Some of them will be presented at the conference.

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